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Two Phases for Compact U(1) Pure Gauge Theory in Three Dimensions

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Abstract

We show that if actions more general than the usual simple plaquette action ($\sim F^2_{\mu\nu}$) are considered, then compact $U(1)$ *pure* gauge theory in three Euclidean dimensions can have two phases. Both phases are confining phases, however in one phase the monopole condensate spontaneously 'magnetizes'. For a certain range of parameters the phase transition is continuous, allowing the definition of a strong coupling continuum limit. We note that these observations have relevance to the 'fictitious' gauge field theories of strongly correlated electron systems, such as those describing high-T_c superconductors.

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As shown by Polyakov[1][2], there is a world of difference between *compact* and *non-compact* $U(1)$ pure gauge theory: in the non-compact case the $U(1)$ gauge transformations (and correspondingly the bare connections A_μ) are valued on the whole real line, while in the compact case they are valued on a circle,[1] and thus allow 'magnetic' monopole configurations. These cause three dimensional compact $U(1)$ pure gauge theory to undergo monopole condensation, resulting in a confined disordered phase for all non-zero lattice spacing[3]. However, effectively only the lowest order kinetic term was considered, corresponding in the naïve continuum limit to $\sim F^2_{\mu\nu}$. In this letter we consider more general Lagrangians for the compact case, forming a complement to the study of three dimensional non-compact pure gauge theory reported in ref.[4]. Indeed in contrast to that case, we find that new continuum limits are reachable with more general Lagrangians.

These continuum limits are formed at the phase transition between the confined disordered phase described above, and an ordered phase in which the monopole condensate spontaneously 'magnetizes'. The magnetized state appears in the regime where the lowest order kinetic term has the 'wrong' sign, leading to vacuum instability in the monopole condensate. Possibly the most interesting physical application of these ideas are to recent theories of strongly correlated electron systems, such as those describing high temperature superconductors[5]–[7]: dynamically generated strongly coupled compact $U(1)$ gauge fields naturally arise in their description of the effectively planar state in these materials. At the microscopic level, an order parameter $\Delta_{\mathbf{ij}} = \langle c^\dagger_{\mathbf{i}\alpha} c_{\mathbf{j}\alpha}\rangle$, where $c^\dagger_{\mathbf{i}\alpha}$ is an electron creation operator of spin α at site $\mathbf{i}$, plays a central rôle; the compact gauge field arises as the phase $\varphi_{\mathbf{ij}}$ of this 'link field'. Since these 'fictitious' $U(1)$ gauge fields are born at the microscopic level without kinetic terms, but instead receive their dynamics through fermionic (i.e. electronic) fluctuations, the lowest order kinetic term can naturally arise with the wrong sign. In the simplest case of just nearest neighbour interactions (and concentrating on the dielectric state with strong on-site repulsion), mean field approximations indicate that three different phases could exist: a 'uniform phase' in which the phases may be chosen so that $\Delta_{\mathbf{ij}} = const.$, a 'molecular crystal phase' in which $\Delta_{\mathbf{ij}} \neq 0$ for only one bond per site, and a 'flux phase' in which $|\Delta_{\mathbf{ij}}| = const.$ but the sum of the phases around an elementary plaquette (called $F^0_{\mu\nu}(\mathbf{x})$ below) equals π [6][7].[2] The present work can be

[1] We have in mind a lattice formulation.

[2] These same approximations generally disfavour the flux phase, but the approximations are not expected to be reliable for determining the energetics[7].

regarded as furnishing a phenomenological Landau Ginzburg description close to the flux–uniform phase transition, which goes beyond the mean field analysis, but in an unrealistic isotropic setting in which also $|\Delta_{\mathbf{ij}}|$ is held fixed and all other quasiparticle excitations are neglected.

In this respect, we note that the gauge invariance of the (low energy) fluctuations ensures that the effective action for the fictitious gauge field is gauge invariant along the Euclidean 'time' direction[3] also, while the compactness of the $U(1)$ gauge group guarantees that monopole configurations (which are instantons of the planar state) are *a priori* allowed. The flux phase precisely corresponds to maximum magnetization of the monopole condensate along the time direction, thus the fate of the flux phase and of the monopole gas are intimately linked. The other quasiparticles have a profound effect on the dynamics of the monopole gas, so that the resulting physics of these instantons is not yet clear[7][8]. The present formulation may help to clarify the situation, if it can be generalised to include the interactions with the other quasi-particles.

The natural order parameter turns out to be the 'magnetic' field $B_\mu(\mathbf{x}) \equiv \frac{1}{2}\varepsilon_{\mu\nu\lambda}F_{\nu\lambda}(\mathbf{x})$. In a cubic lattice regularization, the bare magnetic field corresponds to the plaquette angle

$$\varepsilon_{\mu\nu\lambda}B^0_\lambda(\mathbf{x}) \equiv F^0_{\mu\nu}(\mathbf{x}) = A^0_\mu(\mathbf{i}a) + A^0_\nu(\mathbf{i}a+\hat{\mu}a) - A^0_\mu(\mathbf{i}a+\hat{\nu}a) - A^0_\nu(\mathbf{i}a) \quad ,$$

where $\varphi_{\mathbf{i},\mathbf{i}+\hat{\mu}} \equiv A^0_\mu(\mathbf{i}a)$, a is the lattice spacing, $\mathbf{x} = \mathbf{i}a + \frac{a}{2}(\hat{\mu}+\hat{\nu})$ is centred in the elementary plaquette, and $\hat{\mu},\hat{\nu}$ are unit vectors in the directions μ,ν. Reality, gauge invariance and periodicity ensure that any physically sensible bare action may be written as a bounded single valued function of the plaquette variables: $\cos\big(B^0_\mu\big)$ and $\sin\big(B^0_\mu\big)$.

In gauge theory, it is usual to think of the partition function $\mathcal{Z}$ as defined by a functional integral over the gauge field $A_\mu(\mathbf{x})$. We take a step backwards however, and define $\mathcal{Z}$ as a functional integral over $B_\mu(\mathbf{x})$, together with the constraint $\partial_\mu B_\mu = 0$ inserted as a functional delta function in the path integral:

$$\mathcal{Z}^0_{(non-compact)} = \int \mathcal{D}\mathbf{B}\,\delta[\,\nabla.\mathbf{B}\,]\,\mathrm{e}^{-\frac{1}{g_0^2}S_0[\mathbf{B}]} \quad . \tag{1}$$

Note that the Jacobian for the change of variables is just a constant (in an Abelian gauge theory). The action S_0 will be left general for the moment, except that we will use the fact, mentioned above, that the microscopic Lagrangian densities are bounded. We will

[3] compactified with circumference inversely proportional to the temperature

take them to be normalized so that this bound is of order one; g_0 will thus be analogous to the electromagnetic coupling constant, being small in the usual Gaussian continuum limit. (We will assign the natural geometrical (inverse length) dimension to A_μ, namely $[A] = 1$, and thus $[B] = 2$, so that in three dimensions g_0 has dimension $[g_0] = \frac{1}{2}$).

However in the compact case, we must take account of apparently singular instanton configurations, corresponding to monopoles sitting at positions $\mathbf{x}_s$ with integer charges q_s, which appear as a result of the fact that the phase of the link field (the bare gauge field) is identified under changes of 2π. We have a choice: we can either keep track of the Dirac strings, explicitly recalling that these are invisible to the microscopic Lagrangian when necessary[1][2], or we can remove them by using the Wu-Yang prescription[9], in which case the gauge field may be chosen to be smooth in patches, and identified across the patches by gauge transformations with non-zero winding number, that is $A_\mu(\mathbf{x})$ is regarded as a connection on a non-trivial $U(1)$ bundle over $\mathbb{R}^3 - \{\mathbf{x}_1, \cdots, \mathbf{x}_N\}$. The two representations are physically equivalent but we will assume the Wu-Yang formalism, because it is more convenient for the continuum limit, and also emphasises that the quantization of monopole charge, even in this pure gauge case, is not particular to the lattice. A DeGrand-Toussaint[10] map to the 'physical' bare magnetic field, by adding integer multiples of 2π to ensure $-\pi < B^0_\mu(\mathbf{x}) \leq \pi$, may be regarded as a lattice Wu-Yang prescription, justifying the statement that the two view-points are equivalent. We will be implicitly assuming that such a map has been performed at the lattice level.

We are now ready to consider the changes monopole fluctuations make to the partition function (1). The natural regime to consider, for example in the theories of high-T_c superconductivity, is $g_0\sqrt{a} \sim 1$. In this regime there is a *freely fluctuating* monopole density of order unity monopoles per elementary cube (i.e. per volume a^3), so that the bare monopole charge density $\rho_0(\mathbf{x}) \sim 2\pi \sum_s q_s \delta(\mathbf{x} - \mathbf{x}_s)$ can be assumed to have a continuum limit $\rho(\mathbf{x})$. In this case the partition function is simply given by

$$\mathcal{Z} = \int \mathcal{D}\rho \int \mathcal{D}\mathbf{B}\ \delta[\,\nabla.\mathbf{B} - \rho\,]\, \mathrm{e}^{-\frac{1}{g^2} S[\mathbf{B}]} \quad , \tag{2}$$

which of course we may integrate to give

$$\mathcal{Z} = \int \mathcal{D}\mathbf{B}\ \mathrm{e}^{-\frac{1}{g^2} S[\mathbf{B}]} \quad . \tag{3}$$

We will show later that this partition function is also obtained from the appropriate limit of the dilute instanton gas approximation[1][2]. Further justification can be obtained by considering the simplest (Gaussian) action

$$S = S_{Gaussian} = \frac{1}{2}\int d^3x\, B^2 \quad . \tag{4}$$

Integrating out the $\mathbf{B}$ field in (2) by writing $\mathbf{B} \to \mathbf{B} - \nabla\varphi$, with $\nabla^2\varphi = -\rho$, gives

$$\mathcal{Z} = \mathcal{Z}_{(Gauss)} \int \mathcal{D}\rho\ \exp\left\{-\frac{1}{8\pi g^2}\int d^3x \int d^3y\ \frac{\rho(\mathbf{x})\rho(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\right\} \quad ,$$

where $\mathcal{Z}_{(Gauss)}$ is the Gaussian integral over transverse $\mathbf{B}$ (the photons), analogous to (1). This is nothing but the required continuum limit of the Banks-Myerson-Kogut formulation of the lattice monopole gas[11]. If we substitute (4) in (3), we see that the disordering effect of the monopole condensate has been total: there is no propagation. We have $\langle B_\mu B_\nu\rangle(\mathbf{p}) = g^2\delta_{\mu\nu}$, which should be compared to the dilute monopole plasma result[2]:

$$\langle B_\mu B_\nu\rangle(\mathbf{p}) = g^2\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2+m^2}\right) \quad , \tag{5}$$

confirming that in this case the Debye correlation length $\xi_d = 1/m \sim a$. The Wilson loop expectation value[12]:

$$\exp\{-W[C]\} = \left\langle \exp\left\{i\int_{\mathcal{A}} \mathbf{B}.d\mathbf{s}\right\}\right\rangle \quad , \tag{6}$$

where $\mathcal{A}$ is the minimal area spanning some macroscopic loop, is easily seen (by completing the square) to be $W[C] \sim \mathcal{A}g^2/a$, so that the theory is confining over distances of order the lattice spacing. These are the results that are expected in this regime (e.g. from a strong coupling expansion), as we will further confirm later.

We see that, even if we consider an action which is a general function of the field strength[4] $S = \int d^3x\, V(B^2)$ we still have no dynamics. In this case it is natural to consider a more general action which reintroduces propagation through higher order derivative terms. In the gauge theories of high-T_c superconductivity, such further terms can in any case be expected to be important. Evidently the partition function (3), yields equivalent physics to that of the Heisenberg ferromagnet (viz. $O(3)$ invariant n-vector model), and, close to a phase transition a sufficiently general effective Landau Ginzburg description arises from an action of the form

$$S = \int d^3x\left\{\frac{\kappa}{2}B^2 + \frac{1}{2M^2}(\partial_\mu B_\nu)^2 + \lambda^{(4)}B^4 + \lambda^{(6)}B^6\right\} \quad . \tag{7}$$

A microscopic Lagrangian for which this is the appropriate description is for example:

$$\mathcal{L}_0(\mathbf{x}) = -\kappa_0 \sum_\mu \cos[B^0_\mu(\mathbf{x})] - \frac{1}{M_0^2}\sum_{\mu,\nu} \sin[B^0_\mu(\mathbf{x})]\sin[B^0_\mu(\mathbf{x}+a\hat{\nu})] + \lambda_0^{(4)}\left(\sum_\mu \cos[B^0_\mu(\mathbf{x})]\right)^2 + \lambda_0^{(6)}\left(\sum_\mu \cos[B^0_\mu(\mathbf{x})]\right)^3 \quad .$$

(Here the bare parameters are assumed to be of order unity as explained previously – thus for example we can be sure that the monopole gas is always in the condensed phase since the action (more strictly fugacity) for a single monopole is of order unity. In principle terms containing $\partial_\mu B_\mu$ could appear in (7) even though microscopically they are forbidden, but these terms correspond to furnishing an action for the monopole charge density in (2) and thus to moving away from this deeply confining regime, as we will see later.) Therefore there are two phases.[4] Both phases have a confining monopole condensate, but in one phase the monopole condensate spontaneously magnetizes and $\langle B_\mu(\mathbf{x})\rangle \neq 0$. Physically, it is easy to give a picture of what happens microscopically: For κ sufficiently negative (to overcome quantum fluctuations that renormalize κ to more positive values) the 'energy' (viz. action) of the monopole changes sign so that it becomes favourable to produce monopoles from the vacuum. Simultaneously however, the 'force' between monopoles changes sign so that opposite sign monopoles are actually repelled from each other – polarizing the vacuum. This runaway instability continues until it is balanced by the positive λ interactions (or ultimately by the periodicity of the Lagrangian). At the microscopic level, the Dirac strings significantly reorder the magnetic field, so that different (but physically equivalent) prescriptions for identifying the monopole charges can give very different qualitative pictures of the resulting stable state. The advantage of the version of the DeGrand-Toussaint prescription we have adopted is that it unties these effects and allows a description in terms of the smooth order parameter $B_\mu(\mathbf{x})$.

From (7) we conclude that, deep in the confining regime, for a certain range of parameters the (zero temperature) phase transition is continuous in the universality class of the three dimensional $O(3)$ vector model Wilson fixed point[13]. Outside this range the transition is first order, and at the boundary we have a tricritical point with mean-field critical exponents. Along the continuous phase transition we can define a continuum limit whose Minkowskian continuation corresponds to a non-unitary theory of pseudo-vector glue-balls

[4] Other phase transitions are of course possible (in principle) but would yield only the cubic rotation group (or subgroup thereof) in the continuum limit.

(or rather photon-balls) where the $U(1)$ glue is bound with binding energy of order the cutoff.

The situation becomes more interesting, if we now reduce the coupling constant g_0, moving away from the deeply confining regime. We will see that the physics smoothly changes into that of the dilute monopole gas phase, which we now consider. For $g\sqrt{a} <$ < 1, the semiclassical limit of the dilute instanton gas (about some global minimum field $\langle\mathbf{B}\rangle$) is a good approximation. In the broken phase we shift $\mathbf{B} \mapsto \langle\mathbf{B}\rangle + \mathbf{B}$, where the vacuum expectation value is taken to be independent of $\mathbf{x}$. The *continuum* integration over magnetic fields with monopole singularities may now be written:

$$\mathcal{Z} = \sum_{N=0}^{\infty} \frac{1}{N!} \sum_{\{q_s=\pm1\}} \left(\prod_{s=1}^{N} \int \frac{d^3x_s}{a^3} \right) \zeta^N \int \mathcal{D}\mathbf{B}\ \delta\Big[\nabla.\mathbf{B} - 2\pi{\textstyle\sum_s} q_s \delta(\mathbf{x}-\mathbf{x}_s)\Big]\ \mathrm{e}^{-\frac{1}{g^2}S[\mathbf{B}]} \quad .$$

Here the fugacity ζ is given to good approximation by $\zeta = \mathrm{e}^{-\varepsilon_0/(g^2a)}$, where ε_0/a is the action of one instanton, and ε_0 is a number of order one which depends on the couplings in S and the lattice type (and in the broken phase on $\langle\mathbf{B}\rangle$). This follows by dimensions and the bounds mentioned earlier. We have also restricted the monopole charge to ±1 since the fugacity for higher charges ($\sim \zeta^{q^2}$) is negligable in this limit. (It is worth remarking that the physical monopole charge per unit cell, is bounded by a lattice-type dependent number – which is $|q_s| \leq 2$ for a cubic lattice[10]).

Expressing the functional delta-function as a functional Fourier transform, using an auxiliary field $\chi(\mathbf{x})$, we can perform the sums above and obtain

$$\mathcal{Z}[J] = \int \mathcal{D}(\mathbf{B},\chi)\ \exp\left\{ -\frac{1}{g^2}S[\mathbf{B}] + \int d^3x\ \left[i\chi\nabla.\mathbf{B} + \frac{2\zeta}{a^3}\cos(2\pi\chi) + \mathbf{J}.\mathbf{B} \right] \right\}$$

(up to proportionality constants on $\mathcal{Z}$ which we always ignore). Here we have also introduced a source $\mathbf{J}(\mathbf{x})$ for $\mathbf{B}$. Now it is helpful to split S into the bilinear kinetic term $\frac{1}{2}\int d^3p\, B_\mu(-\mathbf{p})\Delta^{-1}(p)_{\mu\nu}B_\nu(\mathbf{p})$ and interactions $S_{int}[\mathbf{B}]$ (which are order B^3 or higher). Shifting $B_\mu \mapsto B_\mu - ig^2\Delta_{\mu\nu}.\partial_\nu\chi$ we obtain

$$\begin{aligned} \mathcal{Z}[J] = \int \mathcal{D}(B_\mu,\chi)\ \exp\Big\{ & -\frac{1}{2g^2}B_\mu.\Delta^{-1}{}_{\mu\nu}.B_\nu - \frac{1}{g^2}S_{int}[B_\mu - ig^2\Delta_{\mu\nu}.\partial_\nu\chi] \\ & - \frac{g^2}{2}\int d^3x\ \left[\partial_\mu\chi\,\Delta_{\mu\nu}.\partial_\nu\chi - \frac{m^2}{4\pi^2}\cos(2\pi\chi) \right] + J_\mu.\left(B_\mu - ig^2\Delta_{\mu\nu}.\partial_\nu\chi\right) \Big\} \quad , \end{aligned} \tag{8}$$

where we have introduced the Debye mass $m^2 = 8\pi^2\zeta/(g^2a^3)$. Tracing the factors of g, one can see that in this form the theory is manifestly weakly coupled. (For this it is helpful to note that m^2/g^4 is exponentially small in this regime).

If we specialize to the Gaussian action (4), then (8) neatly summarises Polyakov's solution. To see this, note that in this case $\Delta_{\mu\nu} = \delta_{\mu\nu}$ and $S_{int} = 0$; χ is (up to a factor 2π) the Debye-Huckel potential field used in ref.[2]. The equivalence is completely clear if we write $\mathbf{J} = i\nabla\eta + \tilde{\mathbf{J}}$, where η is Polyakov's source for monopole charge density and the transverse photon source satisfies $\nabla.\tilde{\mathbf{J}} = 0$, and then integrate out $\mathbf{B}$ to obtain:

$$\mathcal{Z} = \int \mathcal{D}\chi \; \exp\left\{ \frac{g^2}{2} \int d^3p \, \tilde{J}_\mu(-\mathbf{p}) \left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] \tilde{J}_\nu(\mathbf{p}) \right.$$
$$\left. - \frac{g^2}{2} \int d^3x \left[\left(\partial_\mu[\chi - \eta]\right)^2 - \frac{m^2}{4\pi^2}\cos(2\pi\chi) \right] \right\} \quad .$$

It follows of course that for the Gaussian action one obtains the same results from (8) as obtained in refs.[1][2], namely m is indeed the Debye mass as stated above and defined in eq.(5), and we have confinement: $W[C] \sim mg^2\mathcal{A}$.

Now note that if we put $g\sqrt{a} = const. \ll 1$, then the instanton computation remains valid, but $m \propto 1/a$. At low energies (equivalent to $a \to 0$), this is the deeply confining and disordered regime we discussed previously. We see that the large effective Debye mass ensures that the contributions from the χ field are negligable for the low energy excitations, and the partition function (8) reduces to (3). Also the Gaussian results stated above go over to those deduced from (3) as they should. This provides our final justification for the effective partition function (3).

Now we briefly survey the results one obtains for the general actions such as (7), away from deep confinement. Firstly, it is not hard to show [by e.g. changes of variables on the quadratic parts of (8)] that the effective susceptibility (propagator) Δ_{eff} for the magnetic field, is given generally, to lowest order in g, by:

$$\Delta^{-1}_{eff}(p)_{\mu\nu} = \frac{1}{g^2}\left\{ \Delta^{-1}(p)_{\mu\nu} + \frac{p^\mu p^\nu}{m^2} \right\} \quad . \tag{9}$$

In the symmetric phase (and for small g) we may take $\kappa = 1$ in (7) by a redefinition of g, so that ${\Delta^{-1}}_{\mu\nu} \equiv \delta_{\mu\nu}(1 + p^2/M^2)$. This yields

$$\Delta_{eff}(p)_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{g^2M^2}{p^2 + M^2} + \frac{p^\mu p^\nu}{p^2} \frac{g^2 m^2_{eff}}{p^2 + m^2_{eff}} \quad ,$$

where $m_{eff}^2 = m^2 M^2/(m^2 + M^2)$. This reduces to the Debye formula (5) in the limit $M \to \infty$ as it should, however we see that generally the transverse susceptibility responds according to the mass M of the 'pseudovector glueball' as expected from (7), but the longitudinal susceptibility behaves as a bound state, of the longitudinal parts of the pseudovector excitation and the Debye mass 'scalar glueball', with a mass m_{eff} which is always less than m or M (and greater than $\min[m, M]/\sqrt{2}$).

In the broken phase we take without loss of generality ${\Delta^{-1}}_{\mu\nu} \equiv n_\mu n_\nu + p^2\delta_{\mu\nu}/M'^2$, where $\mathbf{n}$ is the unit vector in the direction $\langle \mathbf{B} \rangle$. Now the susceptibility has three eigen-directions: The transverse magnon (i.e. along the $\mathbf{p} \times \langle \mathbf{B} \rangle$ direction) remains massless, but in the $\langle \mathbf{B} \rangle - \mathbf{p}$ plane two new directions are distinguished with susceptibilities which are no longer simple poles but of the form $2g^2 m'^2_{eff}/S_\pm$, where $S_\pm = m'^2_{eff} + p^2 + \sqrt{p^4 + 2m'^2_{eff}\, p^2 \cos 2\theta + m'^2_{eff}}$. Here θ is the angle between $\mathbf{p}$ and $\langle \mathbf{B} \rangle$ and the effective mass is the equivalent to that of the unbroken phase: $m'^2_{eff} = m^2 M'^2/(m^2 + M'^2)$.

From (9), these leading order changes to the susceptibility can be incorporated by changing the partition function (3) by

$$S[\mathbf{B}] \mapsto S[\mathbf{B}] + \tfrac{1}{2}\xi_d^2 \int d^3x\, (\nabla.\mathbf{B})^2 \quad , \tag{10}$$

but at higher order in g the effective magnetic field action also inherits, from the χ dynamics in (8), non-local changes (the width of the bound state) proportional to factors of $\nabla.\mathbf{B}$.

It would be interesting to understand what effect the change to weak confinement has on the deeply confining phase transition considered earlier (that is assuming the parameters are tuned so that ξ_d diverges with the correlation length). Can it still be continuous, and if so in what universality class? These questions could be addressed within the epsilon expansion[13] starting from (8), although it is not clear that the epsilon expansion should be reliable here. The first corrections to deep confinement, i.e. where $p/m \ll 1$, come from allowing $\nabla.\mathbf{B}$ terms in (7), the correction to the quadratic part being given by (10). [They correspond to furnishing an action for the monopole charge density in (2)]. The effect of these corrections on the phase transition could be investigated by both the epsilon expansion and the derivative expansion[14]. Since we found no continuous phase transition for general actions in non-compact pure gauge $U(1)$ theory[4], it must be that for sufficiently weak confinement the smooth phase transition discussed earlier, disappears. The simplest assumption is that it becomes first order. This implies that the non-compact case also has two phases, with $\mathbf{B}$ being the order parameter, but that the non-compact

case phase transition is always first order. Since mean field theory allows for continuous phase transitions, this means that a Coleman-Weinberg mechanism operates here as in the classic case of scalar QED: fluctuations drive the non-compact transition first order[15]. But another possibility is that for ξ_d sufficiently large a new phase opens up in which the $\mathbf{B}$ field becomes disordered independently of the effects of the monopole plasma (and not therefore unravelable by DeGrand-Toussaint transformations). This possibility was conjectured recently in the context of three dimensional non-compact QED[16]. Presumably in this phase $\langle \mathbf{B} \rangle$ would still vanish, and the relevant order parameter would have to be composite e.g. B^2. This could be investigated by extending the analysis of the pure gauge non-compact case[4] to allow for such a possibility.

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